Published by Institute of Physics Publishing for SISSA/ISAS
Received: *August 21, 2002*
Accepted: *September 25, 2002*

More self-tuning solutions with H_{MNPQ}

Jihn E. Kim and Hyun Min Lee

Department of Physics and Center for Theoretical Physics
Seoul National University, Seoul 151-747, Korea
E-mail: jekim@phyp.snu.ac.kr, minlee@phya.snu.ac.kr

Abstract: We find more self-tuning solutions by introducing a general form for lagrangian of a 3-index antisymmetric tensor field A_{MNP} in the RS II model. In particular, for the logarithmic lagrangian, $\propto \log(-H^2)$, we obtained a closed form weak self-tuning solution.

Keywords: Field Theories in Higher Dimensions, Extra Large Dimensions, Cosmology of Theories beyond the SM.

JHEP09(2002)052

http://jhep.sissa.it/archive/papers/jhep092002052/jhep092002052.pdf

Contents

1. Introduction

Recently, the self-tuning solutions have been attempted toward solutions of the cosmological constant problem. The self-tuning solutions can be broadly classified to

(i) weak self-tuning solutions: this class requires just the existence of the flat space solution in four-dimensional(4D) space-time, and

(ii) strong self-tuning solutions: this class allows only the 4D flat solution without the possibility of 4D curved space solutions.

Of course, a strong self-tuning solution can be considered as a solution of the cosmological constant problem. However, the recent attempts toward a strong self-tuning solution by Kachru et al. [1] has not been successful due to a non-localizable gravity or resurrection of the fine-tuning problem after curing the singularity [2]. It seems that a strong self-tuning solution is difficult to realize at present.

On the other hand, the weak self-tuning solutions are easier to realize. Because the weak self-tuning solutions do not forbid de Sitter or anti de Sitter space solutions, it is necessary to supply an additional principle to choose a flat one out of numerous possibilities. Witten argued that probably the boundary of different phases is chosen [3], and Hawking argued the euclidian quantum gravity gives the maximum probability for the flat universe [4]. We note that the weak self-tuning solution is a big progress. In this regard, note that in 4D a non-vanishing cosmological constant never allows a flat solution, and one has to fine-tune the 4D cosmological constant at zero to have a flat space solution. However, if a weak self-tuning solution is present, then it is a matter of choosing the flat space solution out of numerous possibilities. Witten and Hawking used the 4-form field strenth[1] $H_{\mu\nu\rho\sigma}$ to show the weak self-tuning solution. However, the 4-form field in 4D is not a dynamical field and a weak self-tuning solution cannot be realized in an evolving universe. Even if a flat solution is chosen in the early universe, phase transitions at later epoches can add vacuum energies and can transform the flat solution to curved ones. If the weak self-tuning solution is realized with a dynamical field with an undetermined integration constant(UIC) c which determines the profile of the dynamical field, then addition of vacuum energy can change the profile of the field so that the resulting space-time remains flat. Therefore, it is necessary to have the weak self-tuning solution with a dynamical field. If a dynamical field plays the required role, it is better for it to be a massless scalar so that it affects the whole space-time. Indeed, a weak self-tuning solution having these properties was found [6, 7] in a 5D Randall-Sundrum II-type model [8], using a 5D 4-form field strength H_{MNPQ}. In 5D, the 4-form field strength has one massless scalar. The lagrangian considered is $1/H^2$, where $H^2 = H_{MNPQ}H^{MNPQ}$. At low energy, this lagrangian can be considered as an effective one. The solution found in [6] can be considered at least as the existence proof for the weak-self-tuning solution.

The 4-form field strength H_{MNPQ}, having one dynamical field in 5D and being massless due to the gauge symmetry, might be a key ingredient to the self-tuning solutions. In this paper, therefore, we consider more general functions of H^2 allowing weak self-tuning solutions.

2. General form for lagrangian with H^2 in 5D

Introducing a 3-index antisymmetric tensor field A_{MNP} in the RS II model [8], let us consider a general form of lagrangian with its field strength $H_{MNPQ} = \partial_{[M}A_{NPQ]}$. Then the 5D action reads

$$S = \int d^4x dy \sqrt{-g}\left(\frac{1}{2}R - \Lambda_b + K\left(H^2\right)\right) + \int_{y=0} d^4x\sqrt{-g_4}(-\Lambda_1)\,, \tag{2.1}$$

where $H^2 = H_{MNPQ}H^{MNPQ}$, g and g_4 are 5D and 4D metric determinants, and Λ_b and Λ_1 are bulk and brane cosmological constants. Here we notice that a surface term is needed for the well-defined variation of the action with respect to A_{MNP},

$$S_{\rm surf} = -2\int d^4x dy \partial_M\left(\sqrt{-g}\frac{\partial K(H^2)}{\partial H^2}H^{MNPQ}A_{NPQ}\right). \tag{2.2}$$

Then, the energy-momentum tensor becomes

$$T_{MN} = -\Lambda_b g_{MN} - \frac{\sqrt{-g_4}}{\sqrt{-g}}g_{\mu\nu}\delta^\mu_M\delta^\nu_N\Lambda_1\delta(y) + T^H_{MN}\,, \tag{2.3}$$

where the contribution coming from A_{MNP} is given by

$$T^H_{MN} = K\left(H^2\right)g_{MN} - 8\frac{\partial K(H^2)}{\partial H^2}H_{MPQR}H_N{}^{PQR}\,. \tag{2.4}$$

[1]The 4-form field strength was used earlier in [5].

To obtain a 4D flat solution, let us take the ansatze for the metric and the field strength as

$$ds^2 = \beta^2(y)\eta_{\mu\nu}dx^\mu dx^\nu + dy^2 \,, \tag{2.5}$$

$$H_{\mu\nu\rho\sigma} = \sqrt{-g}\epsilon_{\mu\nu\rho\sigma}f(y)\,, \qquad H_{5\mu\nu\rho} = 0\,. \tag{2.6}$$

Then, the relevant Einstein equations and the field equation are

$$3\left(\frac{\beta'}{\beta}\right)^2 + 3\left(\frac{\beta''}{\beta}\right) = -\Lambda_b - \Lambda_1\delta(y) + K(H^2) + 8\cdot 3!\frac{\partial K(H^2)}{\partial H^2}f^2\,, \tag{2.7}$$

$$6\left(\frac{\beta'}{\beta}\right)^2 = -\Lambda_b + K(H^2)\,, \tag{2.8}$$

and

$$\partial_M\left(\sqrt{-g}\frac{\partial K(H^2)}{\partial H^2}H^{MNPQ}\right) = 0\,, \tag{2.9}$$

where the argument in $K^2(H^2)$ is $H^2 = -4!f^2$. The bulk equations of motion given above, which gives the condition that eq. (2.8) reproduces eq. (2.7) in the bulk or the Bianchi identity(dH=0), requires $f(y) \propto \beta^{-4}$.

3. Self-tuning solution with $\ln(-H^2)$

We can rewrite eq. (2.8) as

$$|\beta'| = \sqrt{-\frac{\Lambda_b}{6}\beta^2 + \frac{\beta^2}{6}K(H^2)}\,, \tag{3.1}$$

with $H^2 = -2\cdot 4!Q\beta^{-8}$, where Q is a positive integration constant of eq. (2.9). In dS and AdS spaces with the curvature λ (> 0 for dS and < 0 for AdS), λ is added in the square bracket of (3.1). For the existence of the self-tuning solution, a necessary condition is that β' should go to zero as β goes to zero [7],

$$\text{Flat condition: } \beta' \longrightarrow 0 \qquad \text{as} \qquad \beta \longrightarrow 0\,, \tag{3.2}$$

which restricts the form of the functional $K(H^2)$. Note that the de Sitter space solution gives a non zero β' as $\beta \to 0$, and anti de Sitter space solution gives a non zero β as $\beta' \to 0$. Indeed, if a solution satisfying this condition is found and it also allows a localizable gravity, it is an acceptable weak self-tuning solution. Therefore, up to an acceptable 4D gravity, this condition becomes the necessary and sufficient condition for the existence of a 4D flat solution. For instance, for a single term with power form of H^2 it was shown that only a negative power of H^2 satisfies such a necessary and sufficient condition [6, 7]. There can exist more solutions satisfying this condition. In this paper, we are interested in infinite series of H^2 which sum up to interesting elementary functions and satisfy the above condition (3.2). In this section, we show another closed form weak self-tuning solution with the logarithmic function. In the dual picture, the bulk lagrangian can be again a logarithmic function.

3.1 Logarithmic function

Let us take $K(H^2)$ as a logarithmic function of H^2 as

$$K(H^2) = V\log\left(-\frac{H^2}{2\cdot 4!}\right), \tag{3.3}$$

where $V>0$ is needed for producing a conventional kinetic term for A_{MNP} as a perturbation around the background solution. Then, it is easy to check that this function satisfies the needed condition because $\beta^2 K(H^2) = V\beta^2(-8\log\beta+\cdots)$ vanishes as $\beta\to 0$.

With this logarithmic function, the Einstein equations become

$$3\left(\frac{\beta'}{\beta}\right)^2 + 3\left(\frac{\beta''}{\beta}\right) = -\Lambda_b - \Lambda_1\delta(y) + V(\log Q - 8\log\beta - 2), \tag{3.4}$$

$$6\left(\frac{\beta'}{\beta}\right)^2 = -\Lambda_b + V(\log Q - 8\log\beta)\,. \tag{3.5}$$

Then, we obtain a 4D flat solution consistent with Z_2 symmetry as

$$\beta(y) = \exp\left[-\frac{\Lambda_b}{8V} + \frac{1}{8}\log\,Q - \frac{V}{3}(|y|+c)^2\right], \tag{3.6}$$

where the argument in the exponent contains a combination

$$\kappa = -\frac{\Lambda_b}{6} + \frac{V}{6}\log Q\,. \tag{3.7}$$

The undetermined integration constant(UIC) c is determined by the boundary condition at the brane

$$\left.\frac{\beta'}{\beta}\right|_{y=0^+} = -\frac{\Lambda_1}{6}\,, \tag{3.8}$$

as

$$c = \frac{\Lambda_1}{4V}\,. \tag{3.9}$$

Therefore, we find that there exist regular flat solutions for an arbitrary set of (Λ_1, V), irrespective of the value of κ. Moreover, the 4D Planck mass becomes finite even for the non-compact extra dimension,

$$\begin{aligned} M_P^2 &= M^3\int_{-\infty}^{\infty} dy\,\beta^2(y) = M^3 e^{2\kappa/v}\int_{-\infty}^{\infty} dy\, e^{-\frac{v}{2}(|y|+c)^2} \\ &= 2M^3 e^{2\kappa/v}\sqrt{\frac{\pi}{2v}}\left(1-\mathrm{erf}\sqrt{\frac{v}{2}}c\right), \end{aligned} \tag{3.10}$$

where $v=\frac{4}{3}V$, and erf denotes the error function

$$\mathrm{erf}\,x = \frac{2}{\sqrt{\pi}}\int_0^x dy\, e^{-y^2}\,. \tag{3.11}$$

3.2 Dual description

Let us consider the above lagrangian in the dual picture. In the dual picture, a brane coupling of the dual scalar field can be introduced easily.

In the dual description of a form field, the equation of motion for a form field is transformed into the Bianchi identity for its dual form field and the Bianchi identity of the form field becomes the equation of motion of the dual form field. Thus, the equation of motion for A_{MNP}, eq. (2.9), becomes the Bianchi identity for the dual scalar field σ,

$$\mathrm{d}\sigma = \ *\mathrm{H}\cdot\frac{\partial K(H^2)}{\partial H^2}\,. \tag{3.12}$$

That is, with a dual transformation such as[2]

$$H_{MNPQ} = -\frac{1}{4!}\sqrt{-g}\epsilon_{MNPQR}\frac{\partial^R\sigma}{(\partial\sigma)^2}\,, \tag{3.13}$$

the 5D action including a brane coupling becomes

$$\begin{aligned} S = & \int d^4x dy\sqrt{-g}\left(\frac{1}{2}R - \Lambda_b - V\left[2+\log\left(2\cdot 4!(\partial\sigma)^2\right)\right]\right) + \\ & + \int_{y=0} d^4x\sqrt{-g_4}(-\Lambda_1 U(\sigma))\,. \end{aligned} \tag{3.14}$$

Therefore, the equation of motion for σ becomes

$$2V\partial_M\left(\sqrt{-g}\frac{\partial^M\sigma}{(\partial\sigma)^2}\right) = \sqrt{-g_4}\Lambda_1\frac{dU}{d\sigma}\delta(y)\,, \tag{3.15}$$

where a non-constant brane coupling $U(\sigma)$ implies that the Bianchi identity for A_{MNP} is not satisfied any more at the brane. On the other hand, the energy-momentum tensor coming from the dual field becomes

$$T^{\sigma}_{MN} = V\left(-[2+\log(2\cdot 4!(\partial\sigma)^2)]g_{MN} + 2\frac{\partial_M\partial_N\sigma}{(\partial\sigma)^2}\right). \tag{3.16}$$

Then, assuming 4D Poincar é invariance as

$$ds^2 = \beta^2(y)\eta_{\mu\nu}dx^\mu dx^\nu + dy^2\,, \qquad \sigma = \sigma(y)\,, \tag{3.17}$$

we obtain the Einstein's equations and the field equation as

$$3\left(\frac{\beta'}{\beta}\right)^2 + 3\left(\frac{\beta''}{\beta}\right) = -\Lambda_b - \Lambda_1 U(\sigma)\delta(y) - V\left(2+\log(2\cdot 4!\sigma'^2)\right)\,, \tag{3.18}$$

$$6\left(\frac{\beta'}{\beta}\right)^2 = -\Lambda_b - V\log(2\cdot 4!\sigma'^2) \tag{3.19}$$

and

$$2V\left(\frac{\beta^4}{\sigma'}\right)' = \beta^4\Lambda_1\frac{dU}{d\sigma}\delta(y)\,. \tag{3.20}$$

[2]The dual description of $1/H^2$ was considered in [9].

Consequently, with $2 \cdot 4!\sigma'^2 = \beta^8/Q$, the bulk solution for the metric is the same as eq. (3.6) in the case without a scalar coupling. But, due to the presence of the scalar coupling, we get different boundary conditions for the metric and the dual field at the brane:

$$\left.\frac{\beta'}{\beta}\right|_{y=0+} = -\frac{\Lambda_1}{6}U(\sigma(0))\,, \tag{3.21}$$

$$\left.\frac{1}{\sigma'}\right|_{y=0+} = \frac{\Lambda_1}{4V}\frac{dU}{d\sigma}(\sigma(0))\,. \tag{3.22}$$

Therefore, we need two consistency conditions, arising from the existence of the brane and the scalar coupling,

$$c = \frac{\Lambda_1}{4V}U(\sigma(0))\,, \tag{3.23}$$

$$\pm\gamma e^{[\frac{4V}{3}c^2+\Lambda_b/(2V)]} = \frac{\Lambda_1}{4V}\frac{dU}{d\sigma}(\sigma(0))\,, \tag{3.24}$$

where $\gamma \equiv \sqrt{2\cdot 4!}$. That is, the condition for the scalar coupling at the brane becomes

$$\frac{1}{U}\frac{dU}{d\sigma}(\sigma(0)) = \pm\frac{\gamma}{c}e^{[\frac{4V}{3}c^2+\Lambda_b/(2V)]}\,, \tag{3.25}$$

plus one of (3.23) and (3.24).

3.3 Curved space solutions

The de Sitter and anti de Sitter space solutions are parametrized by the curvature λ ($\lambda > 0$ for dS and $\lambda < 0$ for AdS). The relevant equations in the dual picture are

$$3\left(\frac{\beta''}{\beta}\right)+\left(\frac{\beta'}{\beta}\right)^2 - 3\lambda\beta^{-2} = -\Lambda_b - U\Lambda_1\delta(y) + V(-8\ln\beta+\ln\tilde{Q}-2)\,, \tag{3.26}$$

$$6\left(\frac{\beta'}{\beta}\right)^2 - 6\lambda\beta^{-2} = -\Lambda_b + V(-8\ln\beta+\ln\tilde{Q})\,, \tag{3.27}$$

where we used $\tilde{Q}$ to represent the charge. In terms of $A(y) = \ln\beta(y)$, eq. (3.27) becomes

$$\frac{dA}{\sqrt{-\frac{\Lambda_b}{6}+\lambda e^{-2A}-\frac{4}{3}VA+\frac{1}{6}V\ln\tilde{Q}}} = dy\,, \tag{3.28}$$

which can be integrated to give, for the case with the Z_2 symmetry,

$$||y| + \text{constant}| = \int^A \frac{dA}{\sqrt{-\frac{\Lambda_b}{6}+\lambda e^{-2A}-\frac{4}{3}VA+\frac{1}{6}V\ln\tilde{Q}}}\,. \tag{3.29}$$

The left-hand side of eq. (3.29) contains an integration constant and λ, which can be formally expressed as $F(|y|;\lambda(\sigma_0),\ldots)$, where $\sigma_0 = \sigma(0)$.[3] If the scalar-brane coupling is

[3]For $\lambda = 0$, integral of eq. (3.29) gives $(3/2V)\sqrt{\kappa^2 - A}$ or we obtain $A = \frac{3}{4}\frac{\kappa^2}{V} - \frac{1}{3}V(|y|+c)^2$ which is identical to eq. (3.6).

JHEP09(2002)052

assumed, for illustration, as $U(\sigma) = e^{b\sigma}$ with a parameter b, we obtain a curved space boundary condition, similar to (3.25) of the flat case, as

$$\frac{1}{\gamma}\exp\left(\frac{3\tilde{\kappa}^2}{V} - \frac{4}{3}V\tilde{c}^2\right)\cdot\sqrt{\frac{\tilde{c}^2}{\tilde{Q}} + \frac{9\lambda}{4V^2\tilde{Q}}e^{-(3\tilde{\kappa}^2/2V)+(2V\tilde{c}^2/3)}} = \frac{1}{b}\,, \tag{3.30}$$

where tilde denote the curved space constants. There is another condition similar to (3.23),

$$\sqrt{\tilde{\kappa}^2 + \lambda e^{-2A(0)} - \frac{4}{3}VA(0)} = \frac{\Lambda_1}{6}e^{b\sigma(0)}\,. \tag{3.31}$$

For $\lambda = 0$, the relation (3.30) reduces to the flat case $(c/\gamma\sqrt{Q})\exp[(3\kappa^2/V) - \frac{4}{3}Vc^2] = 1/b$. We treated b as a parameter. The constant $\tilde{Q}$ is determined by the finite 4D Newton constant. $\tilde{c}$ is an integration constant. There are two boundary conditions (3.30) and (3.31), but the constants to be determined are three: $\tilde{c}, \lambda$ and $\sigma(0)$. Note that $\sigma(0)$ is an additional constant. Anyway, there exist curved space solutions in our case, with a parameter undetermined. It is different from Kachru *et al.* case where their solution imposes a specific value for b such that two equations are consistent only for $\lambda = 0$. To determine λ uniquely in our case, we need another condition.

4. Other self-tuning solutions

It can be shown that there also exist self-tuning solutions for the exponential functions and their some linear combinations such as

$$K(H^2) = V\exp\left(q\frac{2\cdot 4!}{H^2}\right), \quad Vq > 0\,, \tag{4.1}$$

$$K(H^2) = -V\exp\left(p\frac{H^2}{2\cdot 4!}\right), \quad V > 0\,, \qquad \text{and} \;\; p > 0\,, \tag{4.2}$$

$$K(H^2) = -V\tanh\left(r\frac{H^2}{2\cdot 4!}\right), \quad Vr > 0\,, \tag{4.3}$$

$$K(H^2) = V\coth\left(s\frac{H^2}{2\cdot 4!}\right), \quad Vs > 0\,. \tag{4.4}$$

For the 4D flat solution, there exists a limit that $\beta^2 K(H^2)$ goes to zero for $\beta \to 0$ in both cases, which is consistent with eq. (3.1). When we expand the case (4.1) in power series of $1/H^2$, we can obtain a class of general self-tuning solutions previously argued with negative powers of H^2 in ref. [6]. On the other hand, for cases (4.2), (4.3) and (4.4), we find it interesting that there exists a self-tuning solution for the infinite sum of positive powers of H^2, but we know that a finite sum does not allow a solution [7, 10].

Then, to obtain additional self-tuning solutions, with the boundary condition eq. (3.8), we only need to solve one equation, for example, for cases 4.1 and 4.2, as follows:

$$6\left(\frac{\beta'}{\beta}\right)^2 = -\Lambda_b + Ve^{-q\beta^8/Q}\,, \tag{4.5}$$

$$6\left(\frac{\beta'}{\beta}\right)^2 = -\Lambda_b - Ve^{-pQ\beta^{-8}}\,. \tag{4.6}$$

For the case (4.6), we find that there exists a self-tuning solution only for $\Lambda_b < 0$.

The self-tuning functionals $K(H^2)$ considered in the previous section and the current section can be generalized to those with their argument replaced by some polynomial of H^2

$$K(H^2) = V\log\left[\sum_{n=-N_1}^{N_2} a_n\left(-\frac{H^2}{2\cdot 4!}\right)^n\right] \tag{4.7}$$

$$= V\left[\log\left(\sum_{n=-N_1}^{N_2} a_n Q^n \beta^{8(N_2-n)}\right) - 8N_2\log\beta\right], \tag{4.8}$$

where a_n are arbitrary constant coefficients and $N_{1,2}$ are assumed to be arbitrary natural numbers. Likewise, the case with the exponential form can be also generalized to

$$K(H^2) = V\exp\left[\sum_{n=-N_1}^{N_2} a_n\left(-\frac{H^2}{2\cdot 4!}\right)^n\right] \tag{4.9}$$

$$= V\exp\left[\sum_{n=-N_1}^{N_2} a_n Q^n \beta^{-8n}\right], \tag{4.10}$$

where all a_n with $n > 0$ should be negative for $\beta^2 K(H^2)$ to be zero as β vanishes.

5. Conclusion

We obtained more weak self-tuning solutions of the cosmological constant in RS-II type models with H_{MNPQ}. For many cases of functions of H^2, they cannot be obtained as closed forms. But for a limited class of functions of H^2, it was possible to express the solutions in closed forms. In ref. [6], the closed form solution was obtained for the case of $1/H^2$. In section3 of this paper, we obtained a closed form solution for $\log(-H^2)$. It was also shown that the logarithmic lagrangian allows again the logarithmic lagrangian in the dual picture. Without the scalar-brane coupling, we anticipate that the duality symmetry may play some role for stabilizing the logarithmic lagrangian.

With the coupling of the scalar with the brane, it is possible to restrict the form of solutions. For example, in the dual description with the logarithmic function, the conditions become eqs. (3.30) and (3.31). If the effective 4D cosmological constant or the effective 4D curvature λ is a function of the vacuum expectation value of the scalar field σ at $y = 0$, then the condition could determine λ. In this case, one can hope to find a strong self-tuning solution.

Acknowledgments

This work is supported in part by the BK21 program of Ministry of Education, the KOSEF Sundo Grant, and by the Office of Research Affairs of Seoul National University.

References

[1] N. Arkani-Hamed, S. Dimopoulos, N. Kaloper and R. Sundrum, *A small cosmological constant from a large extra dimension*, *Phys. Lett.* **B 480** (2000) 193 [hep-th/0001197]; S. Kachru, M.B. Schulz and E. Silverstein, *Self-tuning flat domain walls in 5d gravity and string theory*, *Phys. Rev.* **D 62** (2000) 045021 [hep-th/0001206].

[2] S. Förste, Z. Lalak, S. Lavignac and H.P. Nilles, *The cosmological constant problem from a brane-world perspective*, *J. High Energy Phys.* **09** (2000) 034 [hep-th/0006139].

[3] E. Witten, *Fermion numbers in Kaluza-Klein theory*, in *Proc. Shelter Island II Conference*, Shelter Island, 1983, R. Jackiw, H. Khuri, S. Weinberg and E. Witten eds., MIT Press, 1985, p. 369.

[4] S.W. Hawking, *The cosmological constant is probably zero*, *Phys. Lett.* **B 134** (1984) 403; E. Baum, *Zero cosmological constant from minimum action*, *Phys. Lett.* **B 133** (1983) 185.

[5] A. Aurilia, H. Nicolai and P.K. Townsend, *Hidden constants: the theta parameter of QCD and the cosmological constant of $N = 8$ supergravity*, *Nucl. Phys.* **B 176** (1980) 509.

[6] J.E. Kim, B. Kyae and H.M. Lee, *A model for self-tuning the cosmological constant*, *Phys. Rev. Lett.* **86** (2001) 4223 [hep-th/0011118].

[7] J.E. Kim, B. Kyae and H.M. Lee, *Self-tuning solution of the cosmological constant problem with antisymmetric tensor field*, *Nucl. Phys.* **B 613** (2001) 306 [hep-th/0101027].

[8] L. Randall and R. Sundrum, *An alternative to compactification*, *Phys. Rev. Lett.* **83** (1999) 4690 [hep-th/9906064].

[9] K.-S. Choi, J.E. Kim and H.M. Lee, *Dual description of brane world cosmological constant with H_{MNPQ}*, *J. Korean Phys. Soc.* **40** (2002) 207 [hep-th/0201055].

[10] J.E. Kim, B. Kyae and H.M. Lee, *Wave function of the radion in the brane background with a massless scalar field and a self-tuning problem*, hep-th/0110103.

JHEP09(2002)052